CHAPTER SIX

LINEAR SEQUENCE

Sequence:

-A sequence is a set of numbers or terms written in a definite order, with a rule or formula for obtaining the terms.

Example(1)
1, 3, 5, 7, 9, 11……………..
- In this given sequence, each term is obtained by adding 2 to the preceding term or the previous term.

Example(2):
1, 3, 9, 27…
In this given sequence, each term is obtained by multiplying the previous term by 3.

The rule of a sequence:

- Each sequence has a rule, and this can be had by making a careful study of the sequence.
- The rule enables us to know or determine any term or other members of the sequence.
- Example(1) 1, 5, 9, 13 …………….. -
For this sequence, the rule is: Any term + 4 = the next term.
- For this reason, the next two terms of this sequence will be 17 and 21.
-Example(2): 3, 9, 27 …….
-The rule is that: Any number multiplied by 3 = the next term.
- For this reason, the next term = 27 × 3 = 81.
- After this term, the next two terms are 81 × 3= 243, and 243 × 3 = 729.
- Example(3): 2, 5, 11, 23 ……………..
- The rule for this sequence is that: (Any term × 2) + 1 = the next term.

- The next three terms are therefore 47, 95 and 191.
- Example(4) 3, 11, 35 …….. -

The rule is that: (Any number × 3) + 2 = the next term.

-The next three terms are therefore 107, 323 and 971.

- Example(5) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots\dots\dots\dots\dots$
- The rule is that: Any number $\times \frac{1}{2}$ = the next term.
- The next two terms are therefore $\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}, and\ \frac{1}{16} \times \frac{1}{2} = \frac{1}{32}\ i.e. \frac{1}{16}$ and $\frac{1}{32}$.
- Example(6): 1, -2, 4, -8, 16, -32 ……………
- The rule is that:(Any term × -2) = the next term.
- The next two terms are (-32 × -2) = 64 and (64 × -2) = -128. i.e. 64 and -128.

  Example(7): 2, 5, 26 …………
- The rule is that:(Any term squared) +1 = the next term.

  Example(8): 25, 20, 15 ……………….
- The rule is that:(Any term – 5) = the next term.

  Example(9): 25, 40, 70 …………
- The rule is that:(any term -5) × 2 = the next term.

  Example(10): {-6, 10, 26 …….}

  The rule is that: any term added to 16 = the next term.

  The next two terms are 42 and 58.

Series:

- When all the terms of a sequence are added together, we get what we call a series.
- For example, consider the sequence 2, 4, 6, 8, 10 ……..
- If the terms of this series are added together, we shall get 2 + 4 + 6 + 8 + 10 ……
- This is what is referred to as a series.
- Since such a series does not contain a fixed number of terms, it is referred to as an infinite series.
- Consider the sequence 2, 4, 8 …….32.

- By adding the terms together, we get the series 2 + 4 + 8 …… + 32.
- Since this series contains a definite number of terms, it is referred to as a finite series.

Types of sequence:

- Two types of sequence shall be considered and these are:

(i) Arithmetic progression (AP), which is also known as linear sequence.

(b) Geometric progression (G.P), which is also known as exponential sequence.

Arithmetic progression:

- In this type, any term differs from the preceding term by a constant known as the common difference.
- This common difference which is represented by d, can either be positive or negative.

  Examples(1) 2, 5, 8, 11 ……………
- In this, the common difference = 5 – 2 = 3 or 8 – 5 = 3.

  Example(2) 3, 13, 23 ……..
- In this, the common difference = 13 – 3 = 10 or 23 – 13 = 10.

  Example (3) -2, - 4, - 6, -8 ……………………
- In this case, d = - 6 – (- 4) = - 6 + 4 = -2.

  - The first term of an A.P is represented by U_1, the second term byU_2 and the third by U_3 and so on.

  - With reference to A.P, the following must be taken notice of:

  U_1= 1st term = a

  U_2 = 2nd term = a + d

  U_3= 3rd term = a + 2d

  U_4= 4th term = a + 3d

  U_n = nth term = a + (n-1)d
- The nth term of an A.P is therefore given by U_n = a+(n-1)d.
- This is the formula for finding any term of a linear sequence.

(Q1) Find the 11th term of a linear sequence of the form 4, 9, 14, 19 …………

Soln:

$a = 4$ and $d = 9 – 4 = 5$.

From $U_n = a + (n - 1)d$

=> $U_{11} = a + (11 – 1)d$

=> $U_{11} = 4 + (11 – 1) \times 5$

=> $U_{11} = 4+5(10) = 4 + 50$

$= 54$.

(Q2) Find the 20th term of the sequence 4, 6, 8, 10, 12 ……

Soln:

$a = 4$ and $d = 6 - 4 = 2$.

Since $U_n = a + (n - 1)d$, then

$U_{20} = 4 + (20 – 1)\ 2$

$U_{20} = 4 + (19) \times 2$

=> $U_{20} = 4 + 2(19)$

=> $U_{20} = 4 + 38$

=> $U_{20} = 42$.

(Q3) Find the 8th term of a linear sequence of the form 47, 42, 37, 32 ………

Soln:

$a = 47$ and $d = 42 – 47 = - 5$.

Since $U_n = a + (n - 1)d$,

=> U_8 = 47 + (8 – 1) (-5)

=> U_8 = 47 + (-5)(8 -1)

=> U_8 = 47 -5(7)

=> U_8 = 47 -35 = 12

(Q4) Show that the 90th term of the sequence 2, 7, 12, 17 ………… is 447.

Soln:

a = 2 and d = 7 – 2 = 5.

From U_n = a + (n -1)d

=> U_{90} = 2 + (90 – 1) 5

=> U_{90} = 2 + 5(89)

=> U_{90} = 2 + 445 = 447.

(Q5) Find the 12th term of an A.P of the form 7, $6\frac{1}{4}, 5\frac{1}{2} \ldots \ldots \ldots \ldots \ldots \ldots ..$

Soln:

a = 7, and d = $6\frac{1}{4}$ - 7 = 6.25 – 7 = - 0.75.

Since U_n = a + (n -1)d

=> U_{12} = 7 + (12 -1)(-0.75)

=> U_{12} = 7 + (11)(-0.75)

=> U_{12} = 7 + (-8.25)

=> U_{12} = 7 – 8.25 = -1.25.

(Q6) Find the linear sequence whose 8th term is 38 and 22nd term is 108.

Soln:

$U_n = a + (n - 1)d$.

The 8th term =

$U_8 = a + (8 - 1)d$

=> $U_8 = a + 7d$

Since the 8th term = 38

=> a + 7d = 38 …………… eqn (1).

Also the 22nd term = 108.

$U_{22} = a + (22-1)d$

=> $U_{22} = a + 21d$.

Since the 22nd term = 108.

=> a + 21d = 108 ………………. eqn (2)

Solving eqn (1) and eqn (2) simultaneously => a = 3 and d = 5.

The sequence = a, a + d, a + 2d, a + 3d …………………..

= 3, 3+5, 3 + 2(5), + 3 + 3(5) …………….

= 3, 8, 13, 18 ……………………..

(Q7 The fourth term of a linear sequence is 19 and the eleventh term is 54.Find the 8th term.

Soln:

$U_n = a + (n - 1)d$

The fourth term =

$U_4 = a + (4 - 1)d$

=> $U_4 = a + 3d$.

But the fourth term = 19

=> $a + 3d = 19$ ……………. eqn (1)

The 11th term = 54.

$U_{11} = a + (11 – 1)d$

=> $U_{11} = a + 10d$.

Since the 11th term = 54

=> $a + 10d = 54$ …………. eqn (2)

Solving eqn (1) and eqn (2) simultaneously => $d = 5$ and $a = 4$.

The 8th term of the sequence is given by $U_8 = a + (8-1) d$

=> $U_8 = a + 7d$ => $U_8 = 4 + 7(5) = 4 + 35 = 39$.

(Q8) The 10th term of a linear sequence is 16 and the common difference is 2. Determine the first term.

Soln:

$n = 10, d = 2, a = ?$

$U_n = a + (n - 1)d$

=> $U_{10} = a + (10 - 1)d$

=> $U_{10} = a + 9d = a + 9(2)$

$= a + 18$.

Since the 10th term = 16

=> $a + 18 = 16$ => $a = 18 – 16 = 2$ => $a = 2$.

The first term = 2.

(Q9) The second and the fourth terms of a linear sequence are 9 and 17 respectively. Find the common difference.

Soln:

The second term = $U_2 = a + (2 - 1)d$

=> $U_2 = a + d$.

Since the second term = 9, => $a + d = 9$ ……………………..eqn (1)

The 4th term = $U_4 = a + (4 – 1)d = a + 3d$.

Since the 4th term = 17

=> $a + 3d = 17$ ……………………………eqn (2)

Solving eqn (1) and eqn (2) simultaneously => $d = 4$.

(Q10) Find the nth term of the following sequences:

(a) 2, 4, 6, 8.
(b) -7, - 4, -1 and 2.

N/B: To find the nth term of an A.P, we use the relation $U_n = a + (n - 1)d$.

Soln:

(a) The first term = a = 2.

The common difference = d = 4 – 2 =2.

Since $U_n = a + (n - 1)d$

=> $U_n = 2 + (n - 1)2$,

=> $U_n = 2 + 2(n - 1)$

=> $U_n = 2 + 2n - 2 = 2 – 2 + 2n = 2n$.

=> the n^{th} term = U_n= 2n.

(b) a = -7 and d = - 4 – (-7) = - 4 + 7 = 3.

From U_n = a + (n – 1)d

=> U_n = -7 + (n -1)3

=> U_n = -7 + 3(n -1),

=> U_n = -7 + 3n – 3

=> U_n = -7 -3 + 3n,

=> U_n = -10 + 3n

=> U_n = 3n – 10,

=> the n^{th} term = 3n – 10.

(Q11) Given the Arithmetic progression 9, 12, 15, 18 ……………

Find (a) the 8^{th} term.

(b) the 9^{th} term.

Soln:

The first term = a = 9.

The common difference = d = 3.

(a) U_n = a + (n -1)d

=> U_8 = 9 + (8 -1)3

=> U_8 = 9 + (7)3,

=> U_8 = 9 + 21 = 30.

(b) the n^{th} term is given by U_n = a + (n -1)d

=> U_n = a + (n -1)d,

=> U_n= 9 + (n – 1) 3

=> U_n= 9 + 3(n – 1),

=> U_n= 9 + 3n – 3

=> U_n= 9 – 3 + 3n,

=> U_n= 6 + 3n

=> the n^{th} term = U_n = 6 + 3n.